Form 3

Annual Report

LaMer, Ehime University

Date (25, February, 2018)

To Director of LaMer

Principle Investigator:

Affiliation Second Institute of Oceanography, SOA

Position Ph.D Candidates

Name in print Ze-Nan Zhu

Include the report on the result of the project/meeting in a separate sheet.

1. Project / Meeting title

Coastal tomographic mapping of nonlinear tidal currents and residual currents

2. Members of project / meeting

Name	Affiliation	Position	Contribution part
PI Ze-Nan Zhu	Second Institute of Oceanography, SOA	Ph.D Candidates	Data processing
Members Xiao-Hua Zhu	Second Institute of Oceanography, SOA	Senior Research Scientist	Beneficial discussion
LaMer Faculty member in charge Xinyu Guo	Ehime University	Professor	Beneficial discussion

3. Contents (please write in separate sheet, A4-size, within 5 pages including figures and tables. Itemize "Title, members' names and affiliations, aim, procedure, result, publication/conference presentation, perspectives in future").

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Title

Coastal tomographic mapping of nonlinear tidal currents and residual currents

Members' names and affiliations

Name	Institution and Department	Employment position	E-mail
Zhu Xiao-Hua	Second Institute of Oceanography, SOA	Senior Research Scientist	xhzhu@sio.org.cn
Xinyu Guo	Faculty member of LaMer	Professor	guoxinyu@sci.ehime-u.ac.jp

Aim

Compared with traditional techniques of measuring the current field, Coastal Acoustic Tomography (CAT) can easily achieve long-term and synchronous measurement over a large area, obtaining the rapidly varying current field in coastal seas. This project aims to carry out a full discussion on the nonlinear tidal dynamics based on CAT data in coastal regions with Prof. Guo Xinyu of Ehime University. In addition, a discussion with Prof. Guo about how to assimilate the CAT data into the ocean model with triangular mesh by using the ensemble Kalman Filter scheme is also necessary.

Procedure

1. The principal investigator (PI) made an oral presentation "A study on assimilation and application of Coastal Acoustic Tomography" and made a briefly introduction of Coastal Acoustic Tomography and its application in coastal regions.
2. The PI carried out further research with Prof. Guo Xinyu on nonlinear tidal currents in Chinese coastal regions. The PI discussed with Prof. Guo about the generation mechanisms of the nonlinear tidal current (i.e. M4 and M6 tides) observed by Coastal Acoustic Tomography.
3. The PI discussed with Prof. Guo about the schemes of assimilating the Coastal Acoustic Tomography data into the ocean model with unstructured grid.

Results

We select a typical coastal regions of China (Zhitouyang Bay) to discuss the characteristics of nonlinear tidal currents. The 15-min mean depth-averaged current data obtained by Coastal Acoustic Tomography (CAT), which covers about 27 hours in Zhitouyang Bay on the western side of the East China Sea, are used to estimate the semidiurnal tidal current (M_2) as well as its first two overtide currents (M_4 and M_6).

To obtain the M_4 and M_6 tidal currents generated by the advection and quadratic bottom friction terms, we carry out the derivation of these terms in the two-dimensional shallow-water equations. These equations can be written as follows:

$$\begin{cases} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv = -g\frac{\partial \eta}{\partial x} - \frac{C_d}{h+\eta}u\sqrt{u^2+v^2} \\ \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu = -g\frac{\partial \eta}{\partial y} - \frac{C_d}{h+\eta}v\sqrt{u^2+v^2} \end{cases} \tag{1}$$

Here, x, y are the horizontal spatial coordinates (positive denoting eastward and northward respectively, unit is m), t is time, u, v are the horizontal velocity components; g is gravitational acceleration (=9.8 m s^{-2}); η is surface elevation; C_d is bottom drag coefficient (=0.0025); h is the water depth; f is Coriolis parameter (=7.27×10^{-5} s^{-1}). Thus, the nonlinear advection terms are $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$ and $u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$, the nonlinear friction terms are $\frac{C_d}{h+\eta}u\sqrt{u^2+v^2}$ and $\frac{C_d}{h+\eta}v\sqrt{u^2+v^2}$.

The tidal velocity components of M_2 (i.e., u_σ and v_σ) can be written as

$$\begin{cases} u_\sigma = U_c(x,y)\cos(\sigma t) + U_s(x,y)\sin(\sigma t) \\ v_\sigma = V_c(x,y)\cos(\sigma t) + V_s(x,y)\sin(\sigma t) \end{cases} \tag{2}$$

Here, $U_c(x,y)$, $U_s(x,y)$, $V_c(x,y)$, $V_s(x,y)$ are harmonic constants of the M_2 tidal current at each spatial point, σ is the M_2 angular frequency.

The M_4 tide is mainly caused by the nonlinear advection terms of the M_2 tidal current (Parker, 1991). Substituting Eq. (2) into Eq. (1), we find that the nonlinear advection terms can be written as follows:

$$\begin{cases} A_0 + B_c\cos(2\sigma t) + B_s\sin(2\sigma t) \\ C_0 + D_c\cos(2\sigma t) + D_s\sin(2\sigma t) \end{cases} \tag{3}$$

Here, $A_0 = \frac{1}{2}\left(U_c\frac{\partial U_c}{\partial x} + V_c\frac{\partial U_c}{\partial y} + U_s\frac{\partial U_s}{\partial x} + V_s\frac{\partial U_s}{\partial y}\right)$; $C_0 = \frac{1}{2}\left(U_c\frac{\partial V_c}{\partial x} + V_c\frac{\partial V_c}{\partial y} + U_s\frac{\partial V_s}{\partial x} + V_s\frac{\partial V_s}{\partial y}\right)$; $B_c = \frac{1}{2}\left(U_c\frac{\partial U_c}{\partial x} + V_c\frac{\partial U_c}{\partial y} + U_s\frac{\partial U_s}{\partial x} + V_s\frac{\partial U_s}{\partial y}\right)$; $B_s = \frac{1}{2}\left(U_c\frac{\partial U_s}{\partial x} + U_s\frac{\partial U_c}{\partial x} + V_c\frac{dU_s}{\partial y} + V_s\frac{\partial U_c}{\partial y}\right)$; $D_c = \frac{1}{2}\left(U_c\frac{\partial V_c}{\partial x} + V_c\frac{\partial V_c}{\partial y} + U_s\frac{\partial V_s}{\partial x} + V_s\frac{\partial V_s}{\partial y}\right)$; $D_s = \frac{1}{2}\left(U_c\frac{\partial V_s}{\partial x} + U_s\frac{\partial V_c}{\partial x} + V_c\frac{\partial V_s}{\partial y} + V_s\frac{\partial V_c}{\partial y}\right)$.

Then we can obtain the currents from the advection terms (i.e., M_{4_cal}) by integrating with respect to time the M_4 angular frequency terms in Eq. (3), as follows:

$$\begin{cases} u_{2\sigma} = \frac{B_0}{2\sigma}\sin(2\sigma t - \alpha_u) \\ v_{2\sigma} = \frac{D_0}{2\sigma}\sin(2\sigma t - \alpha_v) \end{cases} \tag{4}$$

Here, $B_0 = \sqrt{{B_c}^2 + {B_s}^2}$; $D_0 = \sqrt{{D_c}^2 + {D_s}^2}$; $\alpha_u = \arctan\left(\frac{B_s}{B_c}\right)$; $\alpha_v = \arctan\left(\frac{D_s}{D_c}\right)$. The velocities in Eq. (4), having the M_4 angular frequency, are considered to be the predicted M_4 tidal currents. Then, because the predicted M_4 currents are derived from the advection terms, if the predicted M_4 are similar to the M_4 currents measured by CAT, we can confirm that the advection terms make a primary contribution to the generation of M_4 tidal currents, otherwise the M_4 tidal currents may also be generated by other factors.

In the deep area, where water depths are larger than 60 m, M_4 velocity amplitudes measured by CAT agree well with those predicted by the advection terms in the shallow water equations, indicating that M_4 in the deep area is predominantly generated by the advection terms (Figure 1).

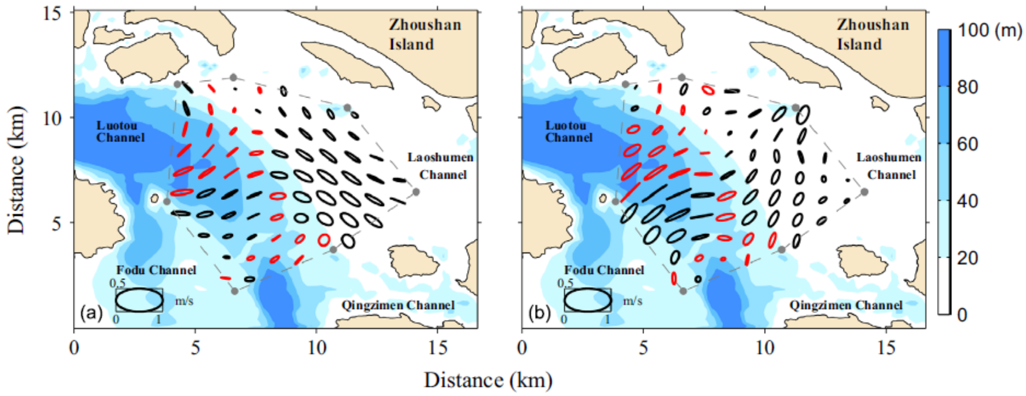

Figure 1. Tidal current ellipses of (a) M_4 and (b) M_{4_cal}. Red lines indicate similar ellipses for M_4 and M_{4_cal}. The area enclosed by the dashed lines indicates the CAT observational region.

On the other hand, the overtide that has three times the M_2 angular frequency (i.e., 3σ) is mainly caused by the friction terms (Fang, 1987; Parker, 1991). The quadratic bottom friction terms of the linear tide can be written in the following form:

$$\begin{cases} \tau_{\mathrm{maj}} = C_d \left| \sqrt{u_{\mathrm{maj}}{}^2 + u_{\mathrm{min}}{}^2} \right| u_{\mathrm{maj}}/h \\ \tau_{\mathrm{min}} = C_d \left| \sqrt{u_{\mathrm{maj}}{}^2 + u_{\mathrm{min}}{}^2} \right| \mathrm{u}_{\mathrm{min}}/h \end{cases} \quad (5)$$

here, τ_{maj} and τ_{min} are the bottom friction along the major and minor axis directions of the linear tide, respectively; u_{maj} and u_{min} are the velocities along the major and minor axis directions of the linear tide, respectively; h is the water depth. Considering $u_{maj} \gg u_{min}$ (Fig.1), Eq. (5) can be written as

$$\begin{cases} \tau_{\mathrm{maj}} = C_d |u_{\mathrm{maj}}| u_{\mathrm{maj}}/h \\ \tau_{\mathrm{min}} = C_d |u_{\mathrm{maj}}| u_{\mathrm{min}}/h \end{cases} \quad (6)$$

Eq. (2) also can be written in the following form:

$$\begin{cases} u_{\mathrm{maj}} = U_{\mathrm{maj}}(x,y) \cos \sigma t \\ u_{\mathrm{min}} = U_{\mathrm{min}}(x,y) \sin \sigma t \end{cases} \quad (7)$$

Here, U_{maj} and U_{min} are the amplitudes along the major and minor axis directions of the linear tide, respectively.

The Fourier expansion of Eq. (6) can be written as follows when we consider only one linear tidal constituent:

$$\begin{cases} \tau_{\mathrm{maj}} = U_{\mathrm{maj}}(x,y)^2 \sum_{m=0,1,2,3,\dots} (-1)^{m+1} \frac{8}{(2m-1)(2m+1)(2m+3)\pi} \cos(2m+1)\sigma t \\ \tau_{\mathrm{min}} = U_{\mathrm{maj}}(x,y) U_{\mathrm{min}}(x,y) \sum_{m=0,1,2,3,\dots} (-1)^{m} \frac{8}{(2m-1)(2m+3)\pi} \sin(2m+1)\sigma t \end{cases} \quad (8)$$

The predicted M_6 tidal currents (i.e., M_{6_cal}) are obtained by setting m=1 and integrating Eq. (8) with respect to time as follows:

$$\begin{cases} u_{3\sigma} = U_{\mathrm{maj}}(x,y)^2 \frac{8C_d}{45h\pi} \cos 3\sigma t \\ v_{3\sigma} = -U_{\mathrm{maj}}(x,y) U_{\mathrm{min}}(x,y) \frac{4C_d}{15h\pi} \sin 3\sigma t \end{cases} \quad (9)$$

The velocities in Eq. (9) having the M_6 angular frequency are considered be the predicted M_6 tide. From the similarity between the measured M_6 and predicted M_6 currents, we can confirm the contribution of quadratic bottom friction to the generation of the M_6 tide.

M_6 measured by CAT and predicted by the nonlinear quadratic bottom friction terms agrees well in areas where water depths are less than 20 m, indicating that friction mechanisms are predominant for generating M_6 in the shallow area (Figure 2).

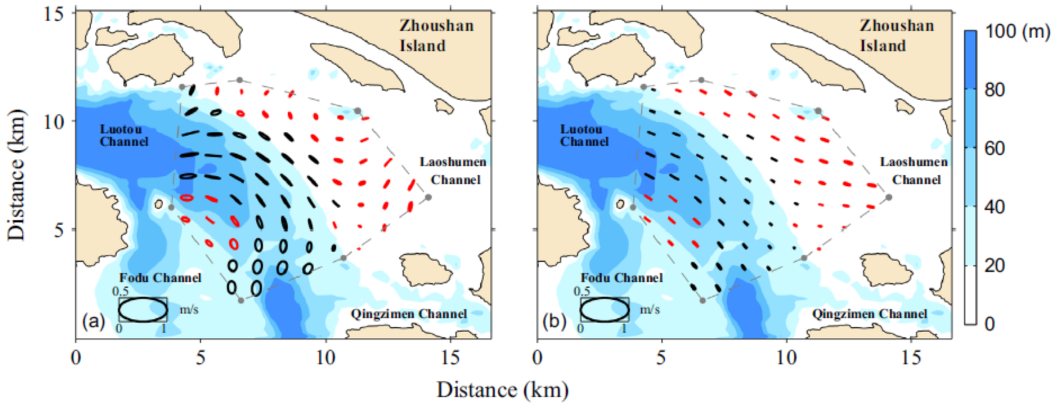

Figure 2 Tidal current ellipses of (a) M_6 and (b) M_{6_cal}. Red lines indicate similar ellipses for M_6 and M_{6_cal}. The area enclosed by the dashed lines indicates the CAT observational region.

Then, we also made the first challenge to assimilate the CAT data into an ocean model with an unstructured triangular grid by using the ensemble Kalman filter scheme (Evensen, 1994).

In order to establish a relationship between CAT data and model states, the relationship between the sound transmission lines and the triangular mesh is described in Figure 3. The original CAT data are travel time differences ($\Delta\tau$) between two acoustic CAT stations. At first, we transfer $\Delta\tau$ to average velocity in the direction along the transmission lines ($\overline{V_{CAT}}$) between the two stations using the equation $\overline{V_{CAT}} = -\frac{{C_0}^2\Delta\tau}{2L}$, where C_0 is the reference sound speed and L is the station-to-station distance. Meanwhile, the average velocity along the transmission line in the ocean model ($\overline{V_{model}}$) can be calculated by $\overline{V_{model}} = \sum_{i=1}^{n}\frac{l_i}{L}(u_i cos\theta + v_i sin\theta)$, where u and v are the eastward and northward ocean model velocity components, respectively; n is the number of triangular grids that are crossed by the sound transmission line; the subscript i indicates the i^{th} triangular grid in the sound transmission line and the corresponding distance is l_i; θ is the direction of the sound transmission line, rotating anticlockwise from due east.

In theory, $\overline{V_{CAT}}$ equals $\overline{V_{model}}$, so the relationship between CAT data and model states can be written as:

$$-\frac{{C_0}^2\Delta\tau}{2L} = \sum_{i=1}^{n}\frac{l_i}{L}(u_i cos\theta + v_i sin\theta) \qquad (10)$$

Thus, the relationship between CAT measurements and state vectors of the simulation can be written as:

$$\begin{pmatrix} -\frac{{C_0}^2\Delta\tau_1}{L_1} \\ -\frac{{C_0}^2\Delta\tau_j}{L_j} \\ \vdots \\ -\frac{{C_0}^2\Delta\tau_m}{L_m} \end{pmatrix} = \begin{pmatrix} \cdots & \frac{l_{1\_1}}{L_1}cos\theta_1 & \frac{l_{1\_1}}{L_1}sin\theta_1 & \cdots & \frac{l_{1\_n_1}}{L_1}cos\theta_1 & \frac{l_{1\_n_1}}{L_1}sin\theta_1 & \cdots \\ \cdots & \frac{l_{j\_1}}{L_j}cos\theta_j & \frac{l_{j\_1}}{L_j}sin\theta_j & \cdots & \frac{l_{j\_n_j}}{L_j}cos\theta_j & \frac{l_{j\_n_j}}{L_j}sin\theta_j & \cdots \\ & \vdots & & & \vdots & & \\ \cdots & \frac{l_{m\_1}}{L_m}cos\theta_M & \frac{l_{m\_1}}{L_m}sin\theta_m & \cdots & \frac{l_{m\_nm}}{L_m}cos\theta_m & \frac{l_{m\_nm}}{L_m}sin\theta_m & \cdots \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ \vdots \\ u_{N_i} \\ v_{N_i} \\ \vdots \\ u_N \\ v_N \end{pmatrix} \qquad (11)$$

where N is the element number in the model mesh; the subscript m denotes the sound transmission line number and j denotes the j^{th} sound transmission line. Thus, the first matrix in the right hand of equation 11 is $\boldsymbol{H}$. Here, we just show some coefficients that are crossed by the transmission lines for example.

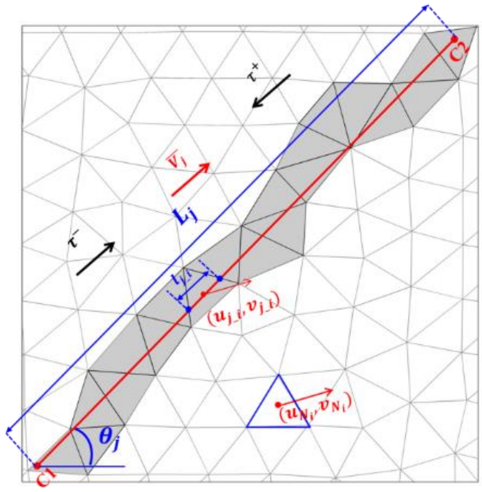

Figure 3. An example of the sound transmission line between C1 and C2 crossing the triangular mesh (gray grids).

Reference

Evensen G., 1994. Sequential data assimilation with a nonlinear quasi-geostrophic model using Monte Carlo methods to forecast error statistics, *J. Geophys. Res.*, 99, C5, 10143-10162.

Fang G. H., 1987. Nonlinear effects of tidal friction. *Acta Oceanologica Sinica*, 6(supp I), 105-122.

Parker B. B., 1991. The relative importance of the various nonlinear mechanisms in a wide range of tidal interactions (review). In: Parker, B. B (Ed.), *Tidal Hydrodynamics*. Wiley New York, USA, pp. 237-268.

Publication/conference presentation

Oral presentation:

Title: A study on assimilation and application of Coastal Acoustic Tomography

Lecturer: Ze-Nan Zhu

Time: August 1, 2017

Location: Ehime University.

Perspectives in future

This project help me a lot to improve the research on nonlinear tidal currents and assimilation scheme based on Coastal Acoustic Tomography data. For the further study, I hope to discuss more with Prof. Guo to improve the assimilation scheme. Meanwhile, I hope to popularize the applications of Coastal Acoustic Tomography in coastal regions.