Title

Analytical solution to the 3D tidal flow with vertically varying eddy viscosity

Members' names and affiliations

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Aim

The tide is often the dominant process in shallow seas and bays; it plays an important role in water mixing and mass transport. Therefore it is essential to understand the tidal motion accurately, especially the spatial characteristics, which is meaningful to oceanography and ocean engineering. The aim of this study is to get the analytical solution to the 3D tidal flow with vertical varying eddy viscosity to improve the studies of the inter-tidal residual circulation in the coastal sea. This can be a basis for further study of the movement of the environment-related substances in the costal water. Another aim of this project is to enhance the cooperation between the College of Environmental Science and Engineering of OUC and the Center for Marine Environmental Studies of Ehime University.

Procedure

The deduction of the analytical solution of the 3D tidal current with vertically varying eddy viscosity in a narrow bay is examined by Prof. Wensheng Jiang and Prof. Xinyu Guo. Its application in the study of the tide-induced residual current is also discussed. During the visit a presentation "The Lagrangian residual current and its application in analytical, numerical and experimental studies in a narrow bay" was given by Prof. Wensheng Jiang in CMES, Ehime University. Prof. Jiang also visited the laboratory of the Center for Marine Environmental Studies accompanied by Prof. Xinyu Guo. They discussed the further cooperation between the two groups.

Results

The 3D tidal current is solved in a semi-enclosed rectangular bay in this study. The *x* and *y* coordinates are along the two horizontal sides of the bay with $x = 0$ being at the open boundary and $x = L$ being at the head of the bay. The lateral boundaries are $y = y_b$ and $y = y_e$. The surface of the still water is set at $z = 0$ and $z = -h$ is the sea bottom. The tidal signal is imposed at the open boundary so the tidal current is the main movement in the area.

Assume the variables to have the following characteristic value: $x_c = \lambda$, $y_c = B$, $z_c = h_c$ and $t_c = 2\pi / \omega_c$, where $\lambda = \sqrt{gh_c} / \omega_c$ is the wave length, h_c is the typical water depth of the sea area and ω_c is the circular frequency of tide, ζ_c is the characteristic value of tidal elevation.

To solve the tidal equations, the double-perturbation is used to expand the tidal equations with regarding to small parameters κ and δ^2, where $\delta = B / \lambda$, $\kappa = \zeta_c / h_c$ and $\beta = \left(\sqrt{\upsilon_c / \omega_c} / h_c\right)^2$ with υ_c being the characteristic value of the vertical eddy viscosity. The zeroth order tidal current are listed below.

$$\nabla \cdot \mathbf{u}_0 = 0 \tag{1}$$

$$\frac{\partial u_0}{\partial t} = -\frac{\partial \zeta_0}{\partial x} + \beta \frac{\partial}{\partial z}\left(\upsilon \frac{\partial u_0}{\partial z}\right) \tag{2}$$

$$0 = -\frac{\partial \zeta_0}{\partial y} \tag{3}$$

At the sea surface, $z = 0$,

$$\begin{gathered} w_0 = \frac{\partial \zeta_0}{\partial t} \\ \frac{\partial (u_0, v_0)}{\partial z} = 0 \end{gathered} \tag{4}$$

At the sea bottom, $z = -h$,

$$\mathbf{u_0} = 0 \tag{5}$$

At the fixed boundary,

$$\begin{gathered} \int_{-h}^{0} v_0 dz = 0 \quad \text{at } y = y_b \text{ and } y_e \\ \int_{-h}^{0} u_0 dz = 0 \quad \text{at } x = L \end{gathered} \tag{6}$$

At the open boundary, $x = 0$,

$$\zeta_0 = \zeta_{open} \tag{7}$$

According to the previous study the vertical eddy viscosity is often of parabolic form, so the expression of the vertical eddy viscosity is assumed as $\upsilon(z) = a_1(z + a_2)(z + a_3)$ in this study. As seen in Fig. 1, υ has the minimum value

R $(0 < R < 1)$ at $z = -h_0$ and attains its maximum value 1 at $z = z_m = -h_0 h_m$, where h_0 is the maximum value of the water depth.

Solutions are presented in a bay with the water depth varying along the transverse direction:

$$N_0 = \frac{\cos(\mu(L-x))}{\cos(\mu L)} \tag{8}$$

$$U_0 = \frac{i\mu \sin(\mu(L-x))}{\cos(\mu L)} q \tag{9}$$

$$V_0 = -\frac{i\mu^2 G \cos(\mu(L-x))}{P_0 \cos(\mu L)} q \tag{10}$$

$$W_0 = \frac{i\mu^2 \cos(\mu(L-x))}{\cos(\mu L)} [-(\frac{\mu^{-2}}{P_0} + \frac{G}{P_0^2}\frac{dP_0}{dy})\int_{-h}^{z} q dz' + \frac{G}{P_0}\int_{-h}^{z} \frac{\partial q}{\partial y} dz'] \tag{11}$$

where N_0, U_0, V_0 and W_0 are amplitude of the water elevation, velocities in the three directions, respectively. The parameters in the above formula are in the following forms:

$\mu^{-2} = -\int_0^1 P_0 dy$ is a constant, $G = -\int_0^y P_0(y')dy' - y\mu^{-2}$ and $P_0 = \int_{-h}^{0} q dz$. q is the solution of a non-homogeneous hypergeometric equation. The results are drawn in Fig. 2.

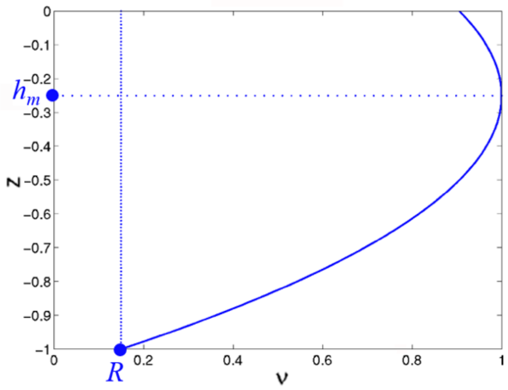

Fig. 1 The profile of the non-dimensional vertical eddy viscosity.

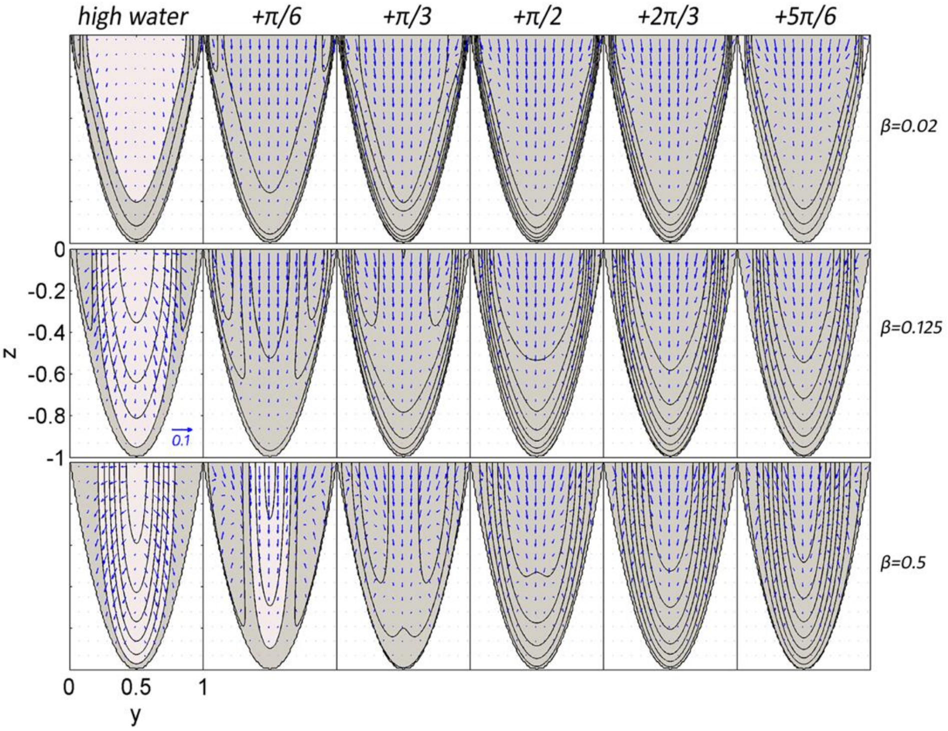

Fig. 2 Velocities at six tidal phases for three different values of β ($R=0.15$, h_m=1/4). The section is located at mid-bay. The leftmost column corresponds to the phase of high water. The axial velocity is negative in the shaded area; lateral and vertical velocities are represented by arrows.

Publication/conference presentation

It is under preparation.

Perspectives in future

The analytical solution of the 3D current with varying eddy viscosity in a narrow bay can be used as a benchmark for the numerical model. It is also a basis for the further acquisition of the tide-induced residual current in the same model bay. It can enhance the understanding of the residual current in a real bay.